This set contains three pages (beginning with this page)
All questions must be answered
Questions 1 and 2 each weigh 25 % while question 3 weighs 50 %. These weights, however, are only indicative for the overall evaluation.

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MONETARY ECONOMICS: MACRO ASPECTS
SOLUTIONS TO JUNE 21 EXAM, 2012

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) The Taylor Principle in monetary policy requires that the central bank raises the interest rate when output increases above its natural rate and lowers the interest rate when output falls below the natural rate.

FALSE. This principle says nothing about how the central bank should respond to movements in the output gap. Instead, the Taylor principle states that the central bank should raise the nominal interest rate when inflation increases. Importantly, this increase should be greater than one-for-one such that the real interest rate increases. (It can be mentioned that this principle secures stability or uniqueness in a wide range of models.)

(ii) Under a nominal interest-rate operating procedure, it is never optimal to take movements in the nominal money supply into consideration when setting the interest rate.

FALSE. If shocks to goods demand and supply cannot be observed, but the nominal money supply shock can, movements in this aggregate can be informative about the unobservable shocks. Therefore it can be optimal to use this information. If money market shocks are predominant, however, the informational content of movements in the nominal money supply becomes limited. Another advantage of responding to the nominal money supply, is that it can circumvent the indeterminacy problem that may arise in models with interest rate pegs.

(iii) In the simple New-Keynesian Phillips curve where only prices are sticky, inflation depends positively on current marginal costs and thereby negatively on the natural rate of output.

TRUE. In this type of model, imperfectly competitive producers set prices as a mark up over marginal costs. As the natural rate of output is driven by productivity shocks, a higher natural rate of output is synonymous with higher productivity, and thus smaller marginal costs. All things equal, prices will be set lower. In the sticky-price setting of the New-Keynesian model, this implies lower inflation.

QUESTION 2:

Monetary policy with a "cash-in-advance" constraint

Consider an economy formulated in discrete time, where the utility of a representative agent is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1, \tag{1}$$

where c_t is real consumption and $u' > 0$, $u'' < 0$. The agent faces the budget constraint

$$\begin{aligned} \omega_t &\equiv f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{m_{t-1} + (1+i_{t-1}) b_{t-1}}{1+\pi_t} \\ &= c_t + k_t + m_t + b_t, \end{aligned} \tag{2}$$

where k_{t-1} is real capital at the end of period $t-1$, f is a production function where $f' > 0$, $f'' < 0$, τ_t denotes real monetary transfers from the government, $0 < \delta < 1$ is the rate of depreciation of capital, m_{t-1} denotes real money holdings at the end of period $t-1$, i_{t-1} is the nominal interest rate on bonds (denoted b_{t-1} in real terms), and π_t is the rate of inflation.

The agent also faces the following cash-in-advance constraint on consumption:

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t. \tag{3}$$

(i) Examine the optimal choices of consumption, capital and real money holdings. For that purpose, show first that the budget constraint (2) can be rewritten as

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + R_t (\omega_t - c_t - k_t - m_t),$$

with $R_t \equiv (1+i_t) / (1+\pi_{t+1})$ being the real interest rate. Use that the agent's optimization problem can be characterized by

$$V(\omega_t, m_{t-1}) = \max \left\{ u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left(c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t \right) \right\},$$

where maximization is over c, k, and m, and where μ_t is the multiplier on (3). Then derive and interpret these necessary optimality conditions:

$$u_c(c_t) = \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t,$$

$$\beta V_\omega(\omega_{t+1}, m_t)\left[f_k(k_t) + 1 - \delta\right] = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

and show that by use of the Envelope theorem one finds

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1+\pi_t}.$$

A Forward (2) one period to get

$$\omega_{t+1} \equiv f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t + (1+i_t) b_t}{1+\pi_{t+1}},$$

and use the definition of the real interest rate to get

$$\omega_{t+1} \equiv f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + R_t b_t.$$

Then use (2) to substitute out $b_t = \omega_t - c_t - k_t - m_t$:

$$\omega_{t+1} \equiv f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + R_t(\omega_t - c_t - k_t - m_t).$$

When the agent's optimization problem can be characterized by

$$V(\omega_t, m_{t-1}) = \max\left\{u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t\left(c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t\right)\right\},$$

where ω_{t+1} is given by the expression just derived, we get the following first-order conditions.

Consumption:

$$u_c(c_t) - \beta R_t V_\omega(\omega_{t+1}, m_t) - \mu_t = 0,$$

or

$$u_c(c_t) = \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t,$$

which states that consumption is optimally chosen to equate the marginal gain (in terms of period-t utility) with the marginal costs. These are the discounted next-period marginal wealth loss of less bond investment and the current liquidity cost of consumption due to the CIA constraint (given $\mu_t > 0$).

Capital:

$$\beta V_\omega(\omega_{t+1}, m_t)[f_k(k_t) + 1 - \delta] - \beta R_t V_\omega(\omega_{t+1}, m_t) = 0,$$

or

$$\beta V_\omega(\omega_{t+1}, m_t)[f_k(k_t) + 1 - \delta] = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

which states that capital is chosen such that its discounted marginal wealth gain (from the marginal product of capital) equals its discounted marginal wealth loss (from lower bond holdings).

Real money holdings:

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

or,

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t)$$

which states that money is chosen such that the discounted marginal gains (in terms of the discounted marginal wealth from more money and the discounted marginal value of money per se) equal the marginal loss (in terms of the discounted marginal wealth loss from lower bond holdings).

In optimum the value function is

$$V(\omega_t, m_{t-1}) = u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left(c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t \right), \tag{*}$$

where it is understood that c_t, k_t and m_t are optimal functions of the state variables ω_t and m_{t-1}. Differentiating (*) w.r.t. ω_t on both sides of (*) gives

$$V_\omega(\omega_t, m_{t-1}) = \beta V_\omega(\omega_{t+1}, m_t) \frac{\partial \omega_{t+1}}{\partial \omega_t},$$

where we have used that we can ignore the partial derivatives of c_t and m_t as these will cancel because c_t and m_t are optimally chosen (satisfy the first-order conditions). From the expression for ω_{t+1}, we readily find that in an optimum $\partial \omega_{t+1} / \partial \omega_t = R_t$ (as we again can ignore the partial derivatives on c_t, k_t and m_t). Hence,

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t)$$

as wanted. Differentiating (*) w.r.t. m_{t-t} on both sides of (*) gives readily

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1+\pi_t},$$

since, again, we again can ignore the partial derivatives on c_t, k_t and m_t, and since ω_{t+1} does not depend on m_{t-1}.

(ii) Let $\lambda_t \equiv V_\omega(\omega_t, m_{t-1})$, and use the results from (i), to obtain an expression for the nominal interest rate, i_t, as a function of μ_{t+1} and λ_{t+1}. Explain this relationship with focus on the role of a binding or non-binding cash-in-advance constraint.

A Combine the first-order condition for real money with the last expression derived using the envelope theorem (to substitute out $V_m(\omega_{t+1}, m_t)$ from the former

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta \mu_{t+1} \frac{1}{1+\pi_{t+1}} = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

Write out the expression for the real interest rate and use the definition of λ_t to get

$$\beta \frac{1}{1+\pi_{t+1}} \lambda_{t+1} + \beta \mu_{t+1} \frac{1}{1+\pi_{t+1}} = \beta \frac{1+i_t}{1+\pi_{t+1}} \lambda_{t+1},$$

which reduces to

$$\lambda_{t+1} + \mu_{t+1} = (1+i_t)\lambda_{t+1},$$

from which we recover

$$i_t = \frac{\mu_{t+1}}{\lambda_{t+1}}.$$

From this we see that a positive nominal interest rate goes hand in hand with a binding cash-in-advance constraint. This is because a positive nominal interest rate is an opportunity cost on holding real money, so in this case households will only hold the money necessary to carry out their transactions. Hence, (3) will hold with equality. A non-binding constraint (i.e., where households hold more money than necessary), can only be possible in an optimum if the nominal interest rate is zero.

(iii) Show formally that monetary policy—here different rates of inflation—has no real effects in steady state. Explain the result. Discuss which variables, on the other hand, will be affected by different long-run inflation rates.

A From

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t)$$

we readily see that the real interest rate in steady state is pinned down by households' discount factor:

$$R^{ss} = \frac{1}{\beta}.$$

Combining this with the first-order condition for capital in steady state gives

$$f_k\left(k^{ss}\right) + 1 - \delta = \frac{1}{\beta}.$$

From this we see that the capital stock, and thus production, are determined independent of monetary factors in steady state. As we have $f\left(k^{ss}\right) = c^{ss} + \delta k^{ss}$, consumption will also be independent of monetary factors. We thus have superneutrality. The reason is, as seen, that capital accumulation is unaffected by different inflation rates, and with capital being the only endogenous production input, the result follows. Different inflation rates will, however, imply different nominal interest rates in the steady state; higher inflation implies higher nominal interest rates to maintain the same real interest rate. In this setting, real money holdings will not change with these differences in nominal interest rate: With the CIA constraint, money will be linked to consumption only (indeed, a quantity-theoretic relationship holds: $c^{ss} = m^{ss}$).

QUESTION 3:

Monetary policy trade offs and commitment policies
Consider the following log-linear "New-Keynesian" model:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{1}$$

$$e_t = \rho_u e_{t-1} + \varepsilon_t, \qquad 0 \leq \rho_u < 1, \tag{2}$$

where π_t is goods price inflation, x_t is the output gap, and e_t is a "cost-push" shock, which is assumed to given by the autoregressive process (2), where ε_t is a mean-zero, serially uncorrelated shock. E_t is the rational expectations operator conditional on all information up to and including period t.

(i) Discuss the micro foundations behind equation (1).

A (1) is a New-Keynesian Phillips Curve, which is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness. Prices are set as a markup over marginal costs, and as the output gap is proportional to marginal costs, it enters positively. The more price rigidity (e.g., the lower a probability of price adjustment under a Calvo price setting scheme), the smaller is κ. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be effective for some periods. The shock e_t captures inefficient fluctuations in inflation not captured by output-gap fluctuations (e.g., exogenous variations in the desired mark up).

(ii) Assume that the monetary authority wants to maximize the utility function

$$U = -\frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda x_t^2 + \pi_t^2\right], \qquad \lambda > 0. \tag{3}$$

Discuss the economic foundations for this utility function.

A This type of function can be derived as the second-order Taylor approximation to the representative household's utility function in a model with monopolistic competition and Calvo-style price rigidities. In the economy, there are welfare losses from firms' monopoly power. Moreover, price rigidities cause losses from aggregate mark-ups being different from the desired markup, and under the Calvo-price structure, staggering cause inefficient dispersion of consumption of various goods. (It is excellent to mention that fiscal measures are assumed to counteract the average monopoly distortion, which would otherwise introduce a term like Λx_t capturing that it would be desirable to have output above the—inefficient—natural rate.) The quadratic terms reflect the costs from fluctuations. Inflation is proportional to the inefficient goods dispersion, and output gap fluctuations are proportional to the fluctuations in the markup gap (that causes inefficient fluctuations in consumption and labor). The parameter λ, which is a function of underlying parameters, measures the relative weight on output gap costs relative to inflation variability costs.

(iii) It is assumed that the authority can commit to policies of the form

$$x_t = \psi_x e_t, \qquad \pi_t = \psi_\pi e_t. \tag{4}$$

Find the optimal values of ψ_x and ψ_π. For this purpose use (2) to show that utility can be written as a function of ψ_x alone:

$$U = -\frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t e_t^2 \left[\lambda \psi_x^2 + \left(\frac{1+\kappa\psi_x}{1-\beta\rho_u}\right)^2\right]$$

A With these policies, expected inflation is given by

$$\mathrm{E}_t \pi_{t+1} = \psi_\pi \mathrm{E}_t e_{t+1} = \psi_\pi \rho_u e_t.$$

Inserting this and (4) into (1) gives

$$\psi_\pi e_t = \beta \psi_\pi \rho_u e_t + \kappa \psi_x e_t + e_t,$$

implying

$$\psi_\pi = \frac{1+\kappa\psi_x}{1-\beta\rho_u}.$$

It thus follows that we can write utility as

$$U = -\frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t e_t^2 \left[\lambda \psi_x^2 + \left(\frac{1+\kappa\psi_x}{1-\beta\rho_u} \right)^2 \right].$$

The first-order condition for optimal ψ_x,

$$\frac{\partial U}{\partial \psi_x} = 0,$$

is

$$-\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t e_t^2 \left[\lambda \psi_x + \kappa \frac{1+\kappa\psi_x}{(1-\beta\rho_u)^2} \right] = 0,$$

or,

$$\lambda \psi_x + \kappa \frac{1+\kappa\psi_x}{(1-\beta\rho_u)^2} = 0,$$

which we can rearrange to obtain

$$\psi_x = -\frac{\kappa}{\kappa^2 + \lambda (1-\beta\rho_u)^2} < 0.$$

This is then used with to give

$$\psi_\pi = \frac{1+\kappa\psi_x}{1-\beta\rho_u} = \frac{\lambda (1-\beta\rho_u)}{\kappa^2 + \lambda (1-\beta\rho_u)^2} > 0.$$

(iv) Under discretionary policymaking, the solutions for the output gap and inflation are given as

$$\begin{aligned} x_t &= -\frac{\kappa}{\kappa^2 + \lambda (1-\beta\rho_u)} e_t, \\ \pi_t &= \frac{\lambda}{\kappa^2 + \lambda (1-\beta\rho_u)} e_t. \end{aligned}$$

Compare how inflation responds to the cost-push shock under the particular commitment policy and discretion. Focus on the relevance of $\rho_u = 0$ versus the case of $\rho_u > 0$ for the comparison.

A Inflation rises with the cost-push shock under either form of policies. When $\rho_u = 0$ we see that there are no difference between commitment and discretion. In the case of $\rho_u > 0$, however, one sees that inflation responds less to a cost-push shock under commitment than discretion. This is because with optimal commitment, expected inflation is affected stronger (rises less). This helps stabilizing inflation, compared to the case of discretion. An excellent answer will note that the difference between the policies be expressed if λ in the commitment solution is replaced by $\lambda^c = \lambda(1 - \beta\rho_u) < \lambda$. Then one can see that the commitment policies is the same as discretionary solutions when the policymaker has a lower relative preference for output gap stabilization, i.e., is "conservative" in the Rogoff sense.

(v) Is commitment of the form (4) always advantageous? Explain.

A Yes, if $\rho_u > 0$, it is always advantageous. As explained above, a stronger reaction to the cost-push shock compared to discretion exploits the expectations channel in monetary policy. Thereby, the inflation-outgup gap trade-off is improved. (If there was no advantage, the two solutions would always coincide since both policies are linear functions of the current shock.)

(vi) Can macroeconomic outcomes be improved relative to those arising under (4)? Explain.

A Yes, in these type of models the form of commitment considered here is a "constrained form" (policy is constrained to be a linear function of the current shock). An improvement is possible (also in the case of $\rho_u = 0$) with the full commitment solution, where the policymaker commits to an optimal policy at time zero. In these models, commitment policies will feature policy inertia, or, history dependence, since a commitment to a prolonged response to a (even temporary) shock affects expectations in a beneficial way. Such policies, however, are not time consistent.